PREPARING FOR CSET MATHEMATICS

FREE SAMPLE

CONSTRUCTED RESPONSE

MULTIPLE CHOICE

CSETMATH

The SMART Way to Study

By Jeff Matthew and Dave Zylstra

Preparing for the CSET Mathematics Sample Book

We at CSETMath want to thank you for interest in Preparing *for the CSET - Mathematics*. This Sample Book is designed to give you an overview of what you will find in the full version of each book by providing two constructed response questions and two multiple choice questions. There are three *Constructed Response* workbooks available:

Preparing for the CSET Mathematics—Subtest I (Test Code 211)
Preparing for the CSET Mathematics—Subtest II (Test Code 212)
Preparing for the CSET Mathematics--Subtest lll (Test Code 213).

There are two *Practice Exam* workbooks available:

Preparing for the CSET Mathematics—Subtest I (Test Code 211)
Preparing for the CSET Mathematics—Subtest II (Test Code 212)

Each *Practice Exam* workbook contains a complete practice exam composed of 35 multiple choice questions and 3 constructed response questions. The *Constructed Response* workbooks contain 20 constructed response questions. All of these workbooks contain detailed solutions designed to prepare you to pass the CSET Mathematics Exam. By completing one of the books you will familiarize yourself with the Subject Matter Requirements at the depth needed to successfully pass the CSET.

This sample book contains similar questions as covered in our most popular books *Subtest I (Test Code 211) and Subtest II (Test Code 212).*

If you found these samples helpful, please visit our website at www.CSETMath.com. The website contains complete descriptions of each book as well as ordering information.

We strive to make CSETMath study materials of the highest quality and provide the most relevant materials, so your input and suggestions would be greatly appreciated. Feel free to contact us at Math@CSETMath.com or visit us on the web at www.CSETMath.com

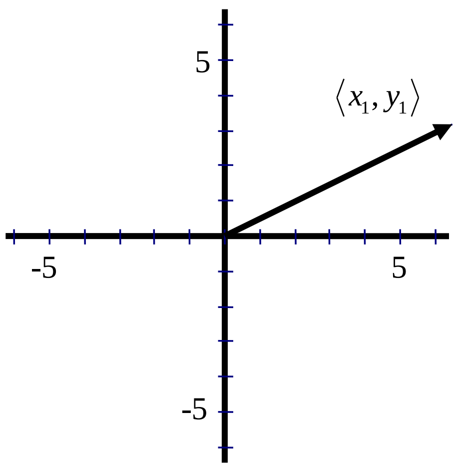
Good luck as you prepare to pass with CSETMath.
The SMART Way to Study

Sample Subtest I

Preparing for the CSET – Mathematics Subtest l Multiple choice

Which of the following sets is a field?
a) {-1, 0, 1}
b) Polynomials
c) 2 X 2 Matrices
d) Complex Numbers

Preparing for the CSET – Mathematics Subtest 1 Constructed Response
Let $\mathbf{v_1} = \langle x_1, y_1 \rangle$ denote a vector in the *xy*-plane with initial point (0, 0) and terminal point (x_1 , y_1) as shown below.



A. Draw $\mathbf{v_1} = \langle 3, -2 \rangle$ and $\mathbf{v_2} = \langle 5, 6 \rangle$ and find their dot products.

B. If **u** lies on the line $y = x$, and **v** lies on the line $y = -x$, show that $\mathbf{u} \bullet \mathbf{v} = 0$.

C. If **u** and **v** lie on perpendicular lines, show that the dot product $\mathbf{u} \bullet \mathbf{v} = 0$.

Sample Subtest II

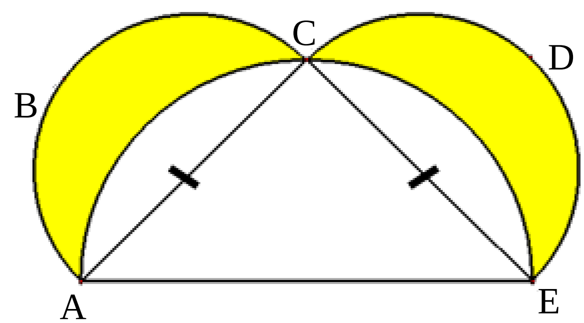
Preparing for the CSET – Mathematics Subtest ll Multiple choice

A bag contains 5 red, 4 black, and 6 blue marbles. If 4 marbles are chosen at random, what is the probability of choosing 2 red and 2 blue?

a) .0110
b) .1099
c) .1333
d) .2667

Preparing for the CSET – Mathematics Subtest ll Constructed Response

Given: ABC, ACE CDE are semicircles, and $\overline{AC} \cong \overline{CE}$ as shown on the diagram at right. Show how the sum of the areas of the shaded regions equals the area of ΔACE



Solution

Preparing for the CSET – Mathematics Subtest l Multiple choice

Which of the following sets is a field?

a) {-1, 0, 1}
b) Polynomials
c) 2 X 2 Matrices
d) Complex Numbers

For an algebraic structure (set of objects – numbers, matrices, etc) to be a field, it must satisfy the six field axioms:

Let x, y, and z be members of the set A

1. The set must be closed under addition and multiplication.
   $x + y$ and $x \cdot y$ are members of the set.
2. Addition and multiplication are commutative for members of the set.
   $x + y = y + x$ and $x \cdot y = y \cdot x$
3. Addition and multiplication are associative for members of the set.
   $(x + y) + z = x + (y + z)$ and $(x \cdot y) \cdot z = x \cdot (y \cdot z)$
4. There exist an additive identity element (0) and a multiplicative identity element (1) such that
   $x + 0 = x$ and $x \cdot 1 = x$
5. There exist an additive and multiplicative inverses such that
   $x + (-x) = 0$ and $x \cdot \frac{1}{x} = 1$
6. Multiplication over addition is distributive
   $x \cdot (y + z) = x \cdot y + x \cdot z$

Let's try out each of the answer choices:

The set {-1, 0, 1} is not a field since it violates the closure property (field axiom #1). 1 + 1 = 2 which is not a member of the set.

The set "Polynomials" violates the multiplicative inverse property (field axiom #5) since a polynomial like $2x^2 + 3$ would have as its inverse $\frac{1}{2x^2 + 3}$. This is not a polynomial since polynomials are not allowed to have variables in the denominator of a fraction.

The set "2 X 2 Matrices" violates the commutative property of multiplication (#2), the associative property of multiplication (#3), and the multiplicative inverse property (#5). Reversing the order in which matrices are multiplied or grouping them differently in multiplication would yield different products. Also, any matrix that has a determinant of 0 does not have a multiplicative inverse.

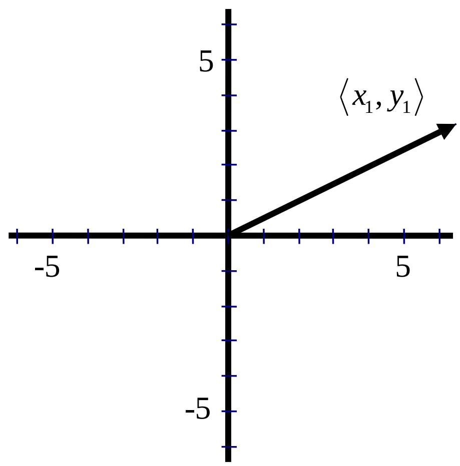
The set "Complex Numbers" (**a + bi**, in which **a** is a Real Number and **i** represents the imaginary number) satisfies all the field axioms, so it is a field.

The answer is D.

Solution

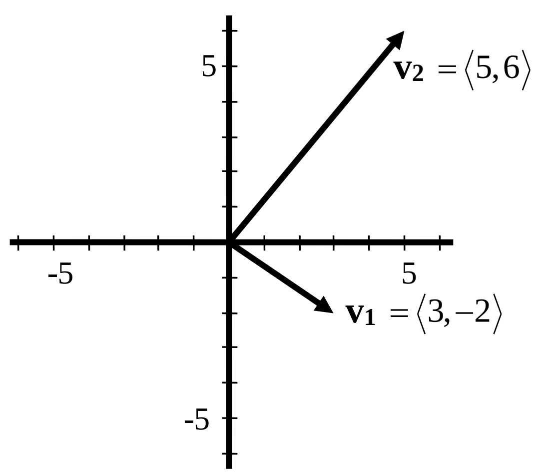
Preparing for the CSET – Mathematics Subtest 1 Constructed Response

Let $\mathbf{v_1} = \langle x_1, y_1 \rangle$ denote a vector in the *xy*-plane with initial point (0, 0) and terminal point (x_1, y_1) as shown below.



A. Draw $\mathbf{v_1} = \langle 3, -2 \rangle$ and $\mathbf{v_2} = \langle 5, 6 \rangle$ and find their dot products.

When a vector is given in "component form", draw the vector as an arrow with the head (terminal point) at the given point, and the tail (initial point) at the origin, as shown below.



To find the dot product $\mathbf{v_1} \bullet \mathbf{v_2}$, multiply the *x*-components together and the *y*-components together, then add the two products.

$$\mathbf{v_1} \bullet \mathbf{v_2} = (3 \cdot 5) + (-2 \cdot 6) = 15 + -12 = 3$$

So the dot product $\mathbf{v_1} \bullet \mathbf{v_2} = 3$

B. If **u** lies on the line $y = x$, and **v** lies on the line $y = -x$, show that **u** • **v** = 0.

Since **u** lies on the line $y = x$, its x- and y- components are equal. Its y-component is whatever its x-component is, so we can say that **u** has the form $\langle x_1, x_1 \rangle$.
Since **v** lies on the line $y = -x$, its x- and y- components are additive inverses.
Its y-component is the negative of whatever its x-component is, so we can say that **v** has the form $\langle x_2, -x_2 \rangle$.

u • **v** = $(x_1 \cdot x_2) + (x_1 \cdot -x_2) = x_1x_2 + -x_1x_2 = 0$
u • **v** = 0

C. If **u** and **v** lie on perpendicular lines, show that the dot product **u** • **v** = 0.

Remember that if two lines are perpendicular, then their slopes are negative reciprocals.
Let vector **u** = $\langle x_1, y_1 \rangle$. Since this names a vector with initial point (0, 0) and terminal point (x_1, y_1), then its slope is $m = \frac{rise}{run} = \frac{y_1 - 0}{x_1 - 0} = \frac{y_1}{x_1}$.

If the slope of **u** is $\frac{y_1}{x_1}$, then the slope of **v** is the negative reciprocal, $-\frac{x_1}{y_1}$.
Since the numerator of the slope represents the y-component, and the denominator represents the x-component, then **v** = $\langle y_1, -x_1 \rangle$.
So, **u** • **v** = $(x_1 \cdot y_1) + (y_1 \cdot -x_1) = x_1y_1 + -y_1x_1 = 0$

Technically, since we do not know how long **v** is, it can be any scalar multiple of $\langle y_1, -x_1 \rangle$, so we should say that **v** = $\langle \text{n}y_1, -\text{n}x_1 \rangle$, but this gives the same result:
u • **v** = $(x_1 \cdot \text{n}y_1) + (y_1 \cdot -\text{n}x_1) = \text{n}x_1y_1 + -\text{n}y_1x_1 = 0$

Solution

Preparing for the CSET – Mathematics Subtest ll Multiple choice

A bag contains 5 red, 4 black, and 6 blue marbles. If 4 marbles are chosen at random, what is the probability of choosing 2 red and 2 blue?

a) .0110
b) .1099
c) .1333
d) .2667

Probability is defined as a fraction in which the numerator is the number of ways that the successful outcome can occur and the denominator is the total number of outcomes.

First find the numerator:

In this problem, success is defined as choosing 2 red and 2 blue marbles. The 2 red marbles are chosen from the 5 red marbles that are available, and the 2 blue marbles are chosen from the 6 blue marbles that are available.

We use Combinations to solve this problem:

${}_nC_r = \dfrac{n!}{r!(n-r)!}$ where **n** is the total items available, **r** is how many we want, and the **!** means "factorial": for example, $6! = 6\cdot5\cdot4\cdot3\cdot2\cdot1 = 720$

For our numerator, we have

$${}_5C_2 \cdot {}_6C_2 = \frac{5!}{2!(5-2)!}\cdot\frac{6!}{2!(6-2)!} = \frac{5!}{2!(3)!}\cdot\frac{6!}{2!(4)!} = 10\cdot15 = 150$$

For the denominator, we also use combinations; but we use the total number of items without regard to what type they are. In other words, we are choosing 4 total marbles from 15 total marbles available.

$${}_{15}C_4 = \frac{15!}{4!(15-4)!} = \frac{15!}{4!(11)!} = 1365$$

So the probability of choosing 2 red and 2 blue marbles is

$$\frac{{}_5C_2 \cdot {}_6C_2}{{}_{15}C_4} = \frac{150}{1365} = .1099$$ which is answer B.

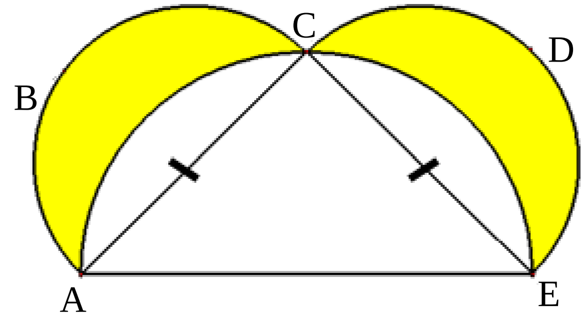
These values could be calculated by multiplying out the factorials, but it is much faster using the ${}_nC_r$ button on the calculator. On a TI-83, enter the value of **n**, press the "MATH" button, arrow over to "PRB" on the display, then arrow down to "${}_nC_r$" on the display. Press "ENTER", then enter the value of **r** and press "ENTER".

The process is similar on a TI-30, except that the ${}_nC_r$ button is above the 8. Enter the value of **n**, press "2nd", then press "8", then enter the value of **r** and press "=".

Solution

Preparing for the CSET – Mathematics Subtest ll Constructed Response

Given: $\overset{\frown}{ABC}$, $\overset{\frown}{ACE}$ $\overset{\frown}{CDE}$ are semicircles, and $\overline{AC} \cong \overline{CE}$ as shown on the diagram at right. Show how the sum of the areas of the shaded regions equals the area of ΔACE



Since ACE is a semicircle, $\angle ACE$ is a right angle because all angles inscribed in a semicircle are right angles. This fact (and given that $\overline{AC} \cong \overline{CE}$) makes ΔABC a right isosceles Δ with $\overline{AE}$ as the hypotenuse. ($\overline{AE}$ is also the diameter of semicircle ACE)

If we let $m\overline{AC}$ = 4x, then the $m\overline{AE} = 4x\sqrt{2}$ by the Pythagorean theorem or isosceles right triangle theorem. (We use 4x so that the radius of the semicircle ABC is 2x and we won't have to worry about fractions.)

Area of semicircle $ABC = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(2x)^2 = \frac{1}{2}\pi(4x^2) = 2\pi x^2$

Area of semicircle $CDE = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(2x)^2 = 2\pi x^2$

Area of semicircle $ACE = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(2x\sqrt{2})^2 = 4\pi x^2$

We now add the two smaller semicircles together to get the larger one.

Area of semicircle ABC + Area of semicircle CDE = Area of semicircle ACE

The shaded area can be described by taking the two secants away from the two smaller semicircles:

Shaded Area = Area of semicircle ABC + Area of semicircle CDE
– (area of segments AC and CE)

But the area of the large semicircle is equal to the area of the 2 smaller semicircles and can be substituted in place of them.

Shaded Area = Area of semicircle ACE – (Area of segments AC and CE)

The area of the triangle can be described by taking the two secants away from the larger semicircle.

Area ΔACE = Area of semicircle ACE – (Area of segments AC and CE)

We see that the Shaded Area and the Area of the triangle are each described the same way, so by substitution,

Shaded Area = Area ΔACE

This Area = ½ bh = ½ $(4x)(4x) = 8x^2$

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